

**AP Calculus BC**  
**Second Quarter Group Project**  
**Designer Polynomials**  
**Due: November 19**

For this project, imagine that you are a calculus instructor needing some “nice” examples of functions for your class. Your purpose is to help the students understand the relationships between polynomial functions and their graphs. In particular, you are interested in quadratic functions,  $x^2 + cx + d$ , cubic functions,  $x^3 + bx^2 + cx + d$ , and quartic functions,  $x^4 + ax^3 + bx^2 + cx + d$ , where  $a$ ,  $b$ ,  $c$ , and  $d$  all have integer values. Thoroughly answer the following questions, giving complete justification for each result. Your final report will be evaluated based on mathematical correctness; clarity of presentation; and correct use of the English language, including structure, grammar and spelling. All work should be neatly written up on another sheet of paper.

- A. Quadratic functions of the form  $f(x) = x^2 + cx + d$
- (1) How many critical points are there for each choice of  $c$  and  $d$ ?
  - (2) Are the critical points maxima, minima, or neither?
  - (3) How should you choose  $c$  and  $d$  to insure that all critical points occur at integer values of  $x$ ?
  - (4) What must the shape of the graph of  $f$  be?
- B. Cubic functions of the form  $g(x) = x^3 + bx^2 + cx + d$
- (5) How many inflection points are there?
  - (6) Produce examples where the inflection points and critical points all occur at integer values of  $x$  where  $g(x)$  has:
    - (a) 0 critical points
    - (b) 1 critical points
    - (c) 2 critical points
  - (7) Write general rules for choosing  $b$ ,  $c$ , and  $d$  to produce families of examples in question 6.
- C. Quartic functions of the form  $h(x) = x^4 + ax^3 + bx^2 + cx + d$
- (8) Choose  $a$ ,  $b$ ,  $c$ , and  $d$  so that  $h(x)$  has 3 critical points at integer values of  $x$ . Produce some general techniques to generate a family of examples with 3 critical points, all at integer values of  $x$ . (Hint: Rather than solving cubic equations to determine if there are 3 integer-valued roots, start with the roots and produce the cubic equation. For example, given the roots 2, 3, and -1, the equation is  $(x - 2)(x - 3)(x + 1) = 0$  or  $x^3 - 4x^2 + x + 6 = 0$ .)

Author: Charles Jones, Grinnell College

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*This should be the first page of your report behind the title page.*

Name	Contribution
1.	
2.	
3.	
4.	

<b>Organization (10 points)</b> Typed, title page, small plastic binder, interesting to read, nice illustrations, questions answered in order, includes all required items, followed instructions	
<b>Communication (10 points)</b> Could be understood by a non-mathematical student, processes used were explained, used complete sentences, answered all the questions	
<b>Mathematics (20 points)</b> All mathematics is correct; no mistakes in calculations	
<b>Group Participation (10 points)</b> Each person contributed his/her part; all persons contributed equally	
<b>Grade</b> <i>(counts as 1 test grade)</i>	<b>/50</b>