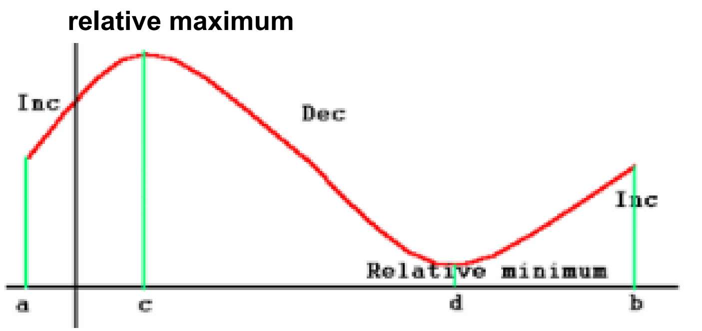
Concavity

The graph of f is concave upward if f ' is increasing on the interval (or the graph of f lies above its tangent lines).

The graph of f is concave downward if f ' is decreasing on the interval (or the graph of f lies below its tangent lines).



Concavity Test

If f '' (x) > 0, then f is concave up.
If f '' (x) < 0, then f is concave down.

Inflection Points

If f has a tangent line at (c, f(c)), the point is called a point of inflection if the concavity of f changes from upward to downward (or vice versa) at that point.

Second Derivative Test

Given: f '(c) = 0 and f '' (c) exists.
Then:

1. If f '' (c) > 0, then f(c) is a relative minimum.
2. If f '' (c) < 0, then f(c) is a relative maximum.
3. If f '' (c) = 0, then the test fails.

Proof:

Suppose f ''(c) > 0

Then f is concave up and lies above its tangent lines.
Since f ' (c) = 0, the tangent line is horizontal at (c, f(c)).
Therefore, f(c) is a minimum.

Suppose f '' (c) < 0
Similar proof

Find concavity, points of inflection, maximum and minimum points, increasing and decreasing intervals for

$y = 2x^4 - 18$. Then graph it.

$2(x^4 - 9) = 0$

$(x^2 - 3)(x^2 + 3) = 0$

$y' = 8x^3 = 0$

$x = 0$

$-$ $+$ y'

0

min $(0, -18)$

$y'' = 24x^2 = 0$

$x = 0$

$+$ $+$ y''

0

Concave up: everywhere

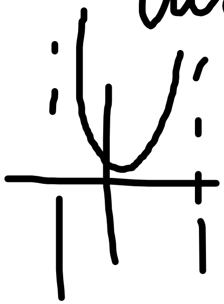
no pts of infl.

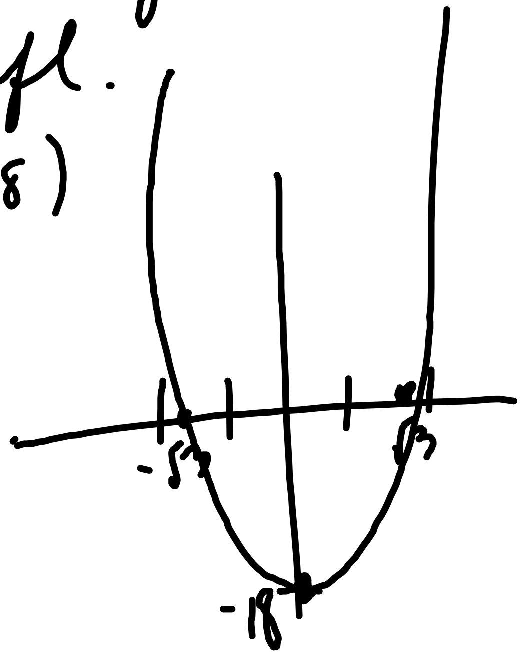
min: $(0, -18)$

no max

inc: $x > 0$

dec: $x < 0$





y': $+$ | $-$ | $+$ at $0 \quad 1 \quad 4$

y'': $-$ | $+$ | $-$ at $0 \quad 6$